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Progressive Taxation and Dynamic Wealth Inequality under Luxury Saving*

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Abstract

When the rich save a larger share of their wealth, wealth inequality grows over time. We ask whether progressive taxation can stop it. The classical theory asks whether a tax makes the distribution more equal at the date it is levied; we ask whether it makes each period's distribution at least as equal as the one before, a property we call *dynamic inequality reduction*. In a two-period model in which myopic agents value wealth for its own sake and saving is a luxury good, the two properties come apart. A progressive income tax reduces inequality at each date, yet no income tax stops it from growing across dates. A wealth tax can, because it also reaches accumulated saving, but it need not: we give an example of a progressive wealth tax that reduces inequality in each period while inequality still grows. Along the way, we show that a progressive commodity tax can reduce inequality in the first period and increase it in the second, and that a wealth tax is more equalizing than an income tax levied through the same marginal-rate progressive schedule. The classical criteria judge a tax by a snapshot, but the problem is a trajectory.

Keywords: wealth taxation, progressive taxation, dynamic inequality reduction, Lorenz dominance, luxury saving, wealth in the utility function

JEL: D31, D63, E21, H23, H24

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1 Introduction

This paper asks whether progressive taxation can stop wealth inequality from rising when saving is a luxury good, that is, when the rich save a larger share of what they have. The classical theory asks whether a tax makes the distribution more equal at the date it is levied. We ask whether it makes the distribution more equal over time: whether each period's distribution is at least as equal as the one before. We call this *dynamic inequality reduction*, and the two questions have different answers. A tax can make the distribution more equal at every date and still leave it growing more unequal from one date to the next.

Wealth inequality has risen across the developed world over the past four decades. In the economies with long-run data, the top wealth share fell after the war and has risen since (Saez and Zucman, 2016). Figure 1 traces the series, and the pattern recurs wherever comparable data can be built (Chancel et al., 2025). The series fluctuate, but every one ends above its own trough. That rise has persisted through decades of progressive taxation, and it is that persistence that brought taxing wealth directly to the center of the debate (Piketty, 2014). The classical theory of inequality-reducing taxation is static: it asks whether a tax leaves the distribution more equal than it would otherwise have been, in the Lorenz sense that every bottom group, the poorest tenth, the poorest fifth, and so on, holds a larger share of the total. A tax can pass that test at every date while inequality keeps rising. Figure 1 does not compare wealth with and without taxes at a given date; it follows wealth under taxation over time, and over time wealth has become more concentrated. That is a statement about the trajectory, and the static test cannot speak to it.

We model a two-period economy of n agents, each receiving a fixed income and earning a common return on what they save. Saving enters utility directly, capturing status and bequest motives, and is a luxury: the share of wealth saved rises with wealth. This property is the paper's sole engine of divergence. Agents maximize utility period by period. One object organizes the analysis, the *magnification*: the rate at which accumulation widens initial wealth differences. It depends on two things, how much of next period's wealth arrives through saving and how sharply saving responds to wealth.

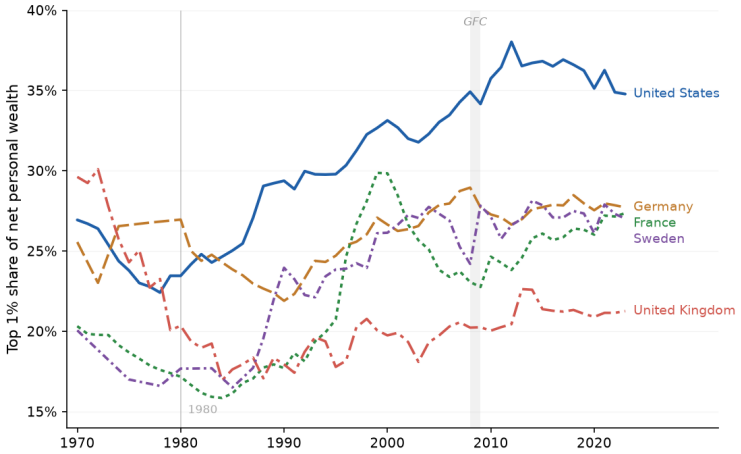


Figure 1: Top 1% share of net personal wealth, 1970–2023.

Notes: Top 1% share of net personal wealth in France, Germany, Sweden, the United Kingdom, and the United States. Source: World Inequality Database (WID.world), series `shweal992j`, percentile `p99p100`. The vertical line marks 1980; the shaded band marks the 2008–09 financial crisis.

Without taxation, the distribution pulls apart: richer agents carry a larger share of their wealth forward, so each period is less equal than the last. No income tax stops this, whatever its rates. It lowers what agents start with but leaves the rate at which they accumulate rising in wealth. A wealth tax is the only instrument considered here that can, because it is levied again on what accumulation has produced. Whether it does depends on how agents save, and we give an example in which a progressive wealth tax leaves the distribution more equal at each date than it would be without the tax, yet the sequence still diverges. Making each date more equal does not make the trajectory more equal.

We get there by first looking at each date on its own. In the first period nobody has saved yet, so our model is the standard static problem and the classical results apply. The second period is different, because the tax now falls on wealth that saving has already spread apart. Income and wealth taxes still make the distribution more equal, provided preferences meet a regularity condition. A progressive commodity tax may not. Taxing consumption makes saving relatively cheaper, so agents save more, and the rich increase their saving by a larger percentage. The second period can then end up less equal than it would be without the tax. Finally, if the same marginal-rate progressive schedule is applied to income in one economy and to wealth in another, the wealth tax

leaves the more equal distribution. It reaches what agents have saved, and savings make up a larger share of what the rich hold.

Saving is a luxury in our model because wealth is valued as an end in itself. The idea goes back to antiquity: accumulation is a way to extend one's existence beyond death through legacy, estate, and standing.[1] Becker (1973, 1975) called this the "immortality project." A bequest meant as a gift and a bequest meant as self-continuation generate the same demand for wealth held rather than spent, so the agent and the dynasty may be treated as one. Economists have carried the idea for half a century. Kurz (1968) first placed wealth in the utility function, and Zou (1994) built it into the Ramsey model as the "spirit of capitalism." Carroll (2000) shows that the saving of the richest households cannot be explained by models in which wealth is only a means to consumption, and Michaillat and Saez (2021) find that treating wealth as an end resolves long-standing puzzles in monetary economics. The motive also predicts the luxury property. Wants for consumption are satiable; the desire for durability is not. As income rises, consumption needs are met first, and the share left for accumulation grows. The data bear this out. Dynan et al. (2004) find that saving rates rise systematically with income, the empirical signature of a luxury good, and Saez and Zucman (2016) trace the rise in wealth concentration to two forces: an upsurge in top incomes and a widening in saving rates. The wealthy also earn higher returns on what they save: Fagereng et al. (2020), tracking Norway's population over twelve years, find that returns rise with net worth across all asset classes. This paper sets differences in returns aside and builds on saving alone, the more conservative case: if luxury saving drives divergence under a common return, return heterogeneity is not needed to produce it.

We also assume that agents choose period by period: each period, they maximize that period's utility, take past saving as given, and carry the result forward. This myopic limit is a benchmark. It is an extreme case, but the alternative is not compelling either. The life-cycle hypothesis (Modigliani and Brumberg, 1954) has agents smooth consumption and die with nothing, which transfers between generations contradict (Kotlikoff and Summers, 1981); the optimization it demands outruns

[1] Representative treatments include Epicurus on kenodoxia (300 BC [2009]), Spinoza's conatus (1677/1994), Lifton's modes of symbolic immortality (1979), and Scheffler's collective afterlife (2013).

plausible cognition (Simon, 1955); and the exponential discounting it assumes is broadly rejected (Strotz, 1955; Laibson, 1997). Myopia also keeps each period's problem static, which is what allows the classical criteria to be applied date by date. Forward-looking agents face a different problem, which Section 5 discusses.

The classical results characterize inequality-reducing taxes at a single date. Jakobsson (1976) and Fellman (1976) show that an income tax reduces inequality in the Lorenz sense for every pre-tax distribution if and only if it is average-rate progressive, and Carbonell-Nicolau (2019) obtains the analogous characterization for commodity taxes in terms of the average tax liability. In these settings incomes are exogenous, there is no saving, and the comparison is between the post-tax and pre-tax distributions at the date of the levy. The first period of our model is an instance of this problem, and the characterizations apply to it directly; for commodity taxes this requires treating saving as an additional, untaxed good, a mapping made precise in Section 3. In the second period the tax falls on a distribution already reshaped by accumulation, and the characterizations must be reexamined. They extend to income and wealth taxes but not to commodity taxes, which alter saving behavior as well as disposable resources.

Dynamic inequality reduction has no counterpart in this literature. The classical criterion compares a taxed distribution with its untaxed counterpart at a date; ours compares the taxed economy with itself across dates. Both are Lorenz comparisons and rest on the same normative ground (Atkinson, 1970), but they are different properties: a tax can satisfy the first at every date and fail the second. The question is also distinct from that of optimal capital taxation (Chamley, 1986; Judd, 1985; Saez and Stantcheva, 2018; Straub and Werning, 2020), which chooses rates to maximize social welfare; we ask instead whether a given schedule can stop the wealth distribution from diverging.

Section 2 develops the framework. Section 3 treats the three instruments within a period, in turn commodity, income, and wealth taxes, and locates the commodity reversal. Section 4 defines dynamic inequality reduction, establishes that the untaxed economy diverges, and asks which instruments arrest it. Section 5 concludes.

2 Model

Consider an economy with $n \geq 2$ individuals who live two periods and receive fixed exogenous income at the beginning of each period. Consumer behavior is described by a continuous utility function, quasi-concave on its domain and strictly so on the interior. The utility function takes k consumption goods and saving as arguments, so $u : \mathbb{R}^{k+1}_{+} \to \mathbb{R}$, characterized by consumption bundles $\boldsymbol{x}$ and saving s, where $\boldsymbol{x} = (x_1, \ldots, x_k)$ is a bundle of $k > 1$ traded goods, and s is the amount saved.

Let $u(\mathbf{x}, s)$ be continuous on $\mathbb{R}^{k+1}_{+}$ and twice continuously differentiable on $\mathbb{R}^{k+1}_{++}$. Additionally, assume $u(\mathbf{x}, s)$ is weakly quasi-concave and monotone in each x_i and in s on $\mathbb{R}^{k+1}_{+}$, with both properties holding strictly on the interior $\mathbb{R}^{k+1}_{++}$. Let $\mathcal{U}$ be the set of all utility functions $u : \mathbb{R}^{k+1}_{+} \to \mathbb{R}$ satisfying the above conditions. Let $S = \mathbb{R}_{+}$ denote the set of feasible saving levels. Write $\mathscr{D} = \mathbb{R}^{k}_{+} \times S$ for the domain and $\mathscr{D}_{++} = \mathbb{R}^{k}_{++} \times S$ for its interior in the consumption coordinates. Then,

$$\mathcal{U} = \left\{ u : \mathscr{D} \to \mathbb{R} \;\middle|\; \begin{array}{ll} (i) & \text{continuous on } \mathscr{D},\ C^2 \text{ on } \mathscr{D}_{++} \\ (ii) & \text{quasi-concave on } \mathscr{D},\ \text{strictly so on } \mathscr{D}_{++} \\ (iii) & \text{weakly monotone in each } x_i \text{ and in } s \text{ on } \mathscr{D},\ \text{strictly so on } \mathscr{D}_{++} \end{array} \right\}$$

Remark 1. *The set $\mathcal{U}$ is non-empty. It contains all additively separable utility functions of the form $u(\boldsymbol{x}, s) = \sum_{j=1}^{k} v_j(x_j) + v_{k+1}(s)$ and all Cobb–Douglas functions $u(\boldsymbol{x}, s) = A s^{\alpha_{k+1}} \prod x_i^{\alpha_i}$ with $A > 0$ and $\alpha_i > 0$, where weak monotonicity and quasi-concavity hold on $\mathbb{R}^{k+1}_{+}$ and both properties hold strictly on $\mathbb{R}^{k+1}_{++}$. CES functions with $\rho < 1$ and Stone–Geary functions under standard parameter restrictions also belong to $\mathcal{U}$.*

Consumers receive fixed exogenous income $z_i \in \mathbb{R}_{++}$ each period and live for two periods. Saving enters the utility function directly, capturing status or bequest motives rather than serving as a vehicle for future consumption. We assume saving satisfies the luxury good condition (Definition 2), a pivotal assumption on which all results depend. Consumers begin period one with nothing saved, that is, $s^0 = 0$.

We model agents as fully myopic, what we term *extreme myopic*: they give no consideration to the future and maximize only current-period utility. As argued in the introduction, this is not a behavioral failure but a rational response to dynamic inconsistency under hyperbolic discounting. The extreme-myopic agent therefore solves each period's problem independently, ignoring the effect of first-period saving on second-period wealth, which yields two independent static problems. For a consumer with income $z \in \mathbb{R}_{++}$, gross return $r \geq 1$, and $\mathbf{p} \in \mathbb{R}^k_{++}$, the extreme-myopic agent solves

$$\max_{\boldsymbol{x}^t \in \mathbb{R}^k_+, s^t \in \mathbb{R}_+} u(\boldsymbol{x}^t, s^t) \quad \text{s.t.} \quad \boldsymbol{p} \cdot \boldsymbol{x}^t + s^t \leq z + r s^{t-1}, \quad t \in \{1, 2\},\ s^0 = 0. \tag{1}$$

Let $w = z + r s^{t-1}$ denote the consumer's wealth in period t, where $s^0 = 0$. By strict quasi-concavity of $u \in \mathcal{U}$ on $\mathbb{R}^{k+1}_{++}$ and Assumption 1, equation (1) has a unique interior solution for every $t \in \{1, 2\}$. The associated Marshallian demand functions are

$$X^u_1(\mathbf{p}, w), \ldots, X^u_k(\mathbf{p}, w), S^u(\mathbf{p}, w).$$

Endowments are distributed across n agents as $\mathbf{z} = (z_1, \ldots, z_n)$ with $z_1 \leq \cdots \leq z_n$. Fix $\bar{z} \in \mathbb{R}_{++}$ and let

$$\mathcal{Z} \;=\; \bigcup_{n \geq 2} \left\{ \mathbf{z} \in \mathbb{R}^n_{++} : z_1 \leq \cdots \leq z_n \leq \bar{z} \right\}$$

denote the set of such distributions with richest endowment at most $\bar{z}$. The bound is a statement about the economy, not about the tax: no population contains unboundedly rich agents, and $\bar{z}$ may be taken as large as the application requires. It is fixed before any schedule is considered, and every quantification over distributions below is over $\mathcal{Z}$.

Definition 1 (Operative range)**.** *Accumulation carries an agent only so far. With endowments bounded by $\bar{z}$, gross return $r \geq 1$, and saving S, the greatest wealth the economy can generate is that of the richest admissible agent after one period of saving out of an untaxed endowment. The* operative range *is the interval of wealth levels the economy can visit,*

$$\mathcal{W} \;=\; \big(0,\ \bar{w}\,\big], \qquad \bar{w} \;=\; \bar{z} + r\, S(\bar{z}),$$

and it is on $\mathcal{W}$ that the conditions below are imposed. A saving function is feasible on $\mathcal{W}$ *if*

$0 < S'(w) < 1$ *there.*

The bound is set by saving, not by taxation. It is the ceiling of the *untaxed* economy, and taxation cannot breach it: a non-negative tax retains no more than wealth, $\rho_T \leq \mathrm{id}$, so post-tax wealth never exceeds pre-tax wealth, and with S increasing the accumulated wealth $z_i + rS(\rho_T(z_i))$ never exceeds $\bar{z} + rS(\bar{z}) = \bar{w}$. Every wealth level any taxed economy produces therefore lies in $\mathcal{W}$, for every $\mathbf{z} \in \mathcal{Z}$ and whatever the schedule, progressive or not. The range is a property of the endowment bound, the return, and the saving motive; the tax merely moves agents around inside it. There is no circularity in the construction: $\bar{w}$ is computed from S on $(0, \bar{z}]$, and feasibility is then imposed on $(0, \bar{w}]$.

Feasibility does two things. The bound $S' < 1$ makes the expansion path $w \mapsto (w - S(w), S(w))$ strictly increasing in both coordinates, so it is rationalized by some utility in $\mathcal{U}$. With luxury saving (Definition 2) it also gives $S(w) < w$, since $S' > S/w$ and $S' < 1$ together put the saving rate below one. Neither is a restriction when S comes from (1), which respects the budget by construction. It binds only where a saving function is posited directly and rationalized backwards. The restriction is not vacuous: constant-elasticity saving $S(w) = cw^\beta$ with $\beta > 1$, the canonical luxury specification, satisfies $S(w) < w$ only below $c^{-1/(\beta-1)}$, so it is feasible on a bounded range and nowhere else.

The three points at which the analysis evaluates a function are thus all in range: the endowment z_i; the post-tax wealth $\rho_T(z_i) \leq z_i$, strictly positive because retention is strictly increasing with $\rho_T(0) = 0$ (Definition 10), so the saving elasticity is never evaluated at zero; and the accumulated wealth $z_i + rS(\rho_T(z_i)) \leq \bar{w}$. What a schedule does above $\bar{w}$ is immaterial, since no agent is ever there.

Definition 2 (Luxury Saving)**.** *Saving is a* luxury good *if the budget share devoted to it is strictly increasing in wealth:*

$$\frac{\partial}{\partial w}\left(\frac{S^u(\mathbf{p}, w)}{w}\right) > 0 \quad \forall w \in \mathcal{W},\ \forall \mathbf{p} \in \mathbb{R}^k_{++}.$$

Equivalently, saving is a luxury good if and only if the marginal propensity to save exceeds the

average propensity to save:

$$\frac{\partial S^u}{\partial w} > \frac{S^u(\mathbf{p}, w)}{w} \quad \forall w \in \mathcal{W},\ \forall \mathbf{p} \in \mathbb{R}^k_{++}.$$

This equivalence follows directly by differentiating $S^u(\mathbf{p}, w)/w$ with respect to w.

Definition 3 (Admissible Preferences)**.** *A utility $u \in \mathcal{U}$ satisfying Definition 2 is* admissible *if its magnification*

$$\Phi(w) \;=\; \lambda(w)\big(\eta(w)-1\big), \qquad \lambda(w) = \frac{r\,\sigma(w)}{1 + r\,\sigma(w)}, \qquad \sigma(w) = \frac{S^u(\mathbf{p}, w)}{w}, \qquad \eta(w) = \frac{w\,S^{u\prime}(\mathbf{p}, w)}{S^u(\mathbf{p}, w)},$$

is non-decreasing on $\mathcal{W}$ for every $\mathbf{p} \in \mathbb{R}^k_{++}$. We write $\mathcal{U}^\star$ for the set of admissible utilities.

The magnification measures how far accumulation widens wealth differences from one period to the next. Since $w^1(z) = z$ and $w^2(z) = z + rS^u(\mathbf{p}, z)$, differentiating gives

$$w^1 \frac{d}{dw^1} \ln w^2 \;=\; 1 + \Phi(w^1),$$

so a one percent difference in period-1 wealth becomes a $(1 + \Phi)$ percent difference in period-2 wealth: $\Phi = 0$ passes differences through unchanged, $\Phi > 0$ widens them.

Wealth reaches the next period through two channels, and two factors divide the effect between them. The part the agent does not save arrives one for one, at elasticity one. The part directed through saving is carried forward at elasticity η, so $\eta - 1$ is the extra magnification that this channel contributes per unit passing through it. How much passes through it is λ, the share of period-2 wealth that saving supplies. Magnification is their product because both are needed. A strong saving response disperses little if saving is a small part of period-2 wealth. A large saving share disperses little if η is near one: the channel carries everything but transmits it nearly proportionally, so rich and poor emerge in the ratio they entered. Dispersion needs both a wide channel and a magnifying one.

Admissibility asks that this magnification not weaken as wealth rises. Luxury saving (Definition 2) makes λ increasing and $\eta > 1$, so $\Phi > 0$ throughout: accumulation always widens wealth differences. Admissibility means that the widening does not decrease as wealth increases. The condition is weaker than requiring η itself to be non-decreasing. Since λ rises with wealth, a falling

η is still admissible if λ rises fast enough to compensate. Only preferences whose saving response weakens faster than the saving share grows are excluded. Because λ depends on the return, Φ and hence $\mathcal{U}^\star$ are defined relative to the r fixed with the environment.

Assumption 1 (Interior Solution)**.** *The solution to* (1) *is interior on the operative range:* $x_i > 0$ *for all* i *and* $s > 0$ *whenever* $\mathbf{p} \in \mathbb{R}^k_{++}$ *and* $w \in \mathcal{W}$.

Interiority is what makes the first-order conditions the operative characterization of demand and, with strict quasi-concavity, gives the unique solution used throughout. It also guarantees $S^u > 0$, without which neither the luxury condition (Definition 2) nor the magnification (Definition 3) could be stated as a ratio. The standard functional forms, Cobb–Douglas, CES, additively separable, and Stone–Geary, satisfy it under strictly positive prices and endowments. Interiority alone does not place a utility in $\mathcal{U}^\star$: the homothetic cases among these forms, Cobb–Douglas and standard CES, fail luxury saving, since a constant saving share cannot be strictly increasing in wealth. The additively separable family does belong, on a bounded range. Its magnification rises while saving is a small part of the budget and turns down only once saving comes to dominate it, so such preferences are admissible below that turning point, and membership depends on where the operative range ends.

Lemma 1 (Wealth Ordering)**.** *If saving is a luxury good (Definition 2), then* $S^u(\mathbf{p}, w)$ *is strictly increasing on* $\mathcal{W}$*, and consequently period-2 wealth* $w^2(z) = z + rS^u(\mathbf{p}, z)$ *is strictly increasing in period-1 wealth* z*. The saving share* $\lambda(w) = r\sigma(w)/(1 + r\sigma(w))$ *is likewise strictly increasing on* $\mathcal{W}$.

Proof. Luxury saving requires $\partial(S^u/w)/\partial w > 0$, whose numerator is $w\,\partial S^u/\partial w - S^u > 0$, so $\partial S^u/\partial w > S^u/w > 0$, the last inequality because $S^u > 0$ at an interior solution (Assumption 1). Hence S^u is strictly increasing in w, and $\partial w^2(z)/\partial z = 1 + r\,\partial S^u/\partial z > 1 > 0$. The same inequality makes the saving rate $\sigma = S^u/w$ strictly increasing, and $x \mapsto rx/(1 + rx)$ is strictly increasing, so λ is too. ∎

Lemma 1 shows that the wealth ordering used throughout, under which agents ranked by period-1 wealth inherit the same ranking in period-2 wealth, follows from luxury saving alone, with no separate normality assumption. In particular saving, being a luxury good, is normal. The monotonicity of λ is what makes the magnification of Definition 3 a product of one factor that always rises and one that need not.

Definition 4 (Lorenz Dominance)**.** *Let $n \geq 2$ and let $\boldsymbol{w} = (w_1, \ldots, w_n)$ and $\boldsymbol{w}' = (w'_1, \ldots, w'_n)$ be wealth distributions arranged in ascending order. Then $\boldsymbol{w}'$* Lorenz dominates *$\boldsymbol{w}$, written $\boldsymbol{w}' \succeq_L \boldsymbol{w}$, if*

$$\frac{\sum_{i=1}^{m} w'_i}{\sum_{i=1}^{n} w'_i} \geq \frac{\sum_{i=1}^{m} w_i}{\sum_{i=1}^{n} w_i} \qquad \forall\, m \in \{1, \ldots, n\},$$

and $\boldsymbol{w}' \succ_L \boldsymbol{w}$ if in addition the inequality is strict for some m. Dominance is a statement of lower *inequality: $\boldsymbol{w}' \succeq_L \boldsymbol{w}$ means every poorest-m group holds at least as large a share of total wealth under $\boldsymbol{w}'$ as under $\boldsymbol{w}$.*

3 Static Reduction and Its Limits

The classical theory characterizes when a tax reduces inequality at a point in time. For a wealth or income tax, average-rate progressivity is necessary and sufficient (Jakobsson, 1976; Fellman, 1976); for a commodity tax, an analogous condition on the average liability (Carbonell-Nicolau, 2019). In the first period of the present model, these characterizations hold as stated: wealth is the endowment, no saving has yet occurred, and the problem is static.

This section asks what becomes of them one period later, after agents have saved. The question is not whether the classical results are correct, but how far they reach: the distribution they compress is the one on which the tax is levied, and by the next period, that distribution has been remade by accumulation. Because saving is a luxury, the remaking is not neutral. A tax that compresses period-1 wealth alters how much each agent carries forward and through which channel, so its verdict in period 2 is a separate question with a separate answer.

Three instruments are considered, and one object organizes all three. Each asks what the tax does to the magnification Φ of Definition 3, the rate at which accumulation widens wealth differences. An income tax leaves Φ intact and relocates the agent on it, at lower wealth; a wealth tax does the same and then levies a second time on what saving has produced; a commodity tax does not relocate the agent but changes relative prices and alters how much they save at any given wealth. That difference is what separates the instruments here. For an income or wealth tax, progressivity lowers the wealth; it cannot place agents where accumulation disperses more sharply than where they began (Propositions 2 and 3) For a commodity tax, Proposition 1 exhibits a preference and a progressive tax that compresses the first period and widens the second. The criterion that governs a single date extends to the next, then, but only for the instruments that leave the magnification alone.

Definition 5 (Static Inequality Reduction)**.** *Let* $\mathbf{y}^t$ *denote the period-t post-tax wealth distribution under a given tax and* $\mathbf{w}^t$ *the period-t wealth distribution in the untaxed economy. The tax achieves* static inequality reduction in period *t if the former Lorenz dominates the latter for every distribution of endowments:*

$$\mathbf{y}^t(\mathbf{z}) \succeq_L \mathbf{w}^t(\mathbf{z}) \qquad \forall\, \mathbf{z} \in \mathcal{Z}.$$

Each subsection below specifies $\mathbf{y}^t$ *for its instrument.*

3.1 Commodity Tax

In each period, a consumer's wealth is $w = z + rs^{t-1}$, comprising exogenous income z and the proceeds of prior saving. Under taxation, the consumer optimizes over post-tax wealth.

Definition 6 (Commodity Tax)**.** *A commodity tax schedule is a vector* $\boldsymbol{\tau} = (\tau_1, \ldots, \tau_k) \in \mathbb{R}^k_+$*, where* τ_l *denotes the tax per unit of good* l *consumed.*

Under commodity tax $\boldsymbol{\tau}$, the effective price vector becomes $\mathbf{p}^{\boldsymbol{\tau}} = \mathbf{p} + \boldsymbol{\tau}$. For $z \in \mathbb{R}_{++}$, $r \geq 1$, $\mathbf{p} \in \mathbb{R}^k_{++}$, and $\boldsymbol{\tau} \in \mathbb{R}^k_+$, the extreme-myopic agent solves

$$\max_{\boldsymbol{x}^t \in \mathbb{R}^k_+, s^t \in \mathbb{R}_+} u(\boldsymbol{x}^t, s^t) \quad \text{s.t.} \quad \mathbf{p}^{\boldsymbol{\tau}} \cdot \boldsymbol{x}^t + s^t \leq z + rs^{t-1}, \quad t \in \{1, 2\}, \quad s^0 = 0. \tag{2}$$

By strict quasi-concavity of $u \in \mathcal{U}$ and Assumption 1, this problem has a unique interior solution for every $t \in \{1, 2\}$, with associated Marshallian demand functions

$$X_1^u(\mathbf{p}^{\boldsymbol{\tau}}, w), \ldots, X_k^u(\mathbf{p}^{\boldsymbol{\tau}}, w), S^u(\mathbf{p}^{\boldsymbol{\tau}}, w).$$

Definition 7 (Luxury Commodity)**.** *Good i is a* luxury commodity *if its budget share is strictly increasing in wealth:*

$$\frac{\partial}{\partial w}\left[\frac{p_i^{\tau} X_i^u(\mathbf{p}^{\boldsymbol{\tau}}, w)}{w}\right] > 0 \quad \forall\, (\mathbf{p}^{\boldsymbol{\tau}}, w) \in \mathbb{R}^k_{++} \times \mathcal{W} \tag{3}$$

Definition 8 (Progressive Commodity Tax)**.** *Fix $u \in \mathcal{U}$ and $\mathbf{p} \in \mathbb{R}^k_{++}$. A commodity tax $\boldsymbol{\tau}$ is* $(u, \mathbf{p})$-progressive *if the average tax liability is non-decreasing in wealth on the operative range:*

$$\frac{\partial}{\partial w}\left[\frac{\sum_{l=1}^k \tau_l X_l^u(\mathbf{p}^{\boldsymbol{\tau}}, w)}{w}\right] \geq 0 \qquad \forall\, w \in \mathcal{W}.$$

Remark 2. *Definition 8 provides a primitive characterization of the inequality-reducing condition in Theorem 1 of Carbonell-Nicolau (2019). Specifically, expanding the derivative in Definition 8 yields*

$$\sum_{l=1}^k \tau_l \frac{\partial X_l^u(\mathbf{p}^{\boldsymbol{\tau}}, w)}{\partial w} \geq \frac{\sum_{l=1}^k \tau_l X_l^u(\mathbf{p}^{\boldsymbol{\tau}}, w)}{w},$$

which is precisely condition (A1) of Carbonell-Nicolau (2019). Average-rate progressivity of the commodity tax liability is therefore necessary and sufficient for a commodity tax to be inequality reducing at the date it is levied, in the sense of their Theorem 1. In period 1, where wealth is the endowment and no saving has yet occurred, this settles the question. [2] *The rest of this subsection concerns period 2, where saving has intervened.*

Under a commodity tax we specialize the wealth distributions of Definition 5 to explicit form. Fix $\mathbf{p} \in \mathbb{R}^k_{++}$, a commodity tax $\boldsymbol{\tau} \in \mathbb{R}^k_{+}$, and an agent with income z. Under the extreme-myopic protocol each period is a static problem.

[2] Their setting has no saving, so the result applies here only after saving is treated as a good. Augment the bundle with saving as a $(k+1)$-th good, $\tilde{\mathbf{x}} = (x_1, \ldots, x_k, s)$, priced at $\tilde{\mathbf{p}}^{\boldsymbol{\tau}} = (p_1^{\tau}, \ldots, p_k^{\tau}, 1)$, which is strictly positive since $\boldsymbol{\tau} \geq 0$ and $\mathbf{p} \gg 0$. The extreme-myopic agent's first-period problem is then the static $(k+1)$-good program $\max u(\tilde{\mathbf{x}})$ subject to $\tilde{\mathbf{p}}^{\boldsymbol{\tau}} \cdot \tilde{\mathbf{x}} \leq z$, of the form Carbonell-Nicolau (2019) study, and $u \in \mathcal{U}$ meets their conditions. A commodity tax leaving saving untaxed is the augmented vector $\tilde{\boldsymbol{\tau}} = (\tau_1, \ldots, \tau_k, 0)$; the saving good bears a zero rate and so contributes nothing to either side of their characterizing inequality, which therefore holds for the augmented system exactly when it holds for $\boldsymbol{\tau}$ on the k consumption goods. The augmentation adds no restriction, and their Theorem 1 applies to $\boldsymbol{\tau}$ as stated.

With no tax the agent faces $\mathbf{p}$ in both periods. Period-1 wealth is the endowment, $w^1(z) = z$; the agent saves $S^u(\mathbf{p}, z)$ and carries into period 2 the wealth

$$w^2(z) \;=\; z + rS^u(\mathbf{p}, z). \tag{4}$$

Under the tax the agent faces $\mathbf{p}^{\boldsymbol{\tau}}$ and pays the commodity liability in each period, on what they buy out of that period's wealth. In period 1 the budget $\mathbf{p}^{\boldsymbol{\tau}} \cdot \mathbf{x}^1 + s^1 = z$ gives saving $S^u(\mathbf{p}^{\boldsymbol{\tau}}, z)$ and liability $\boldsymbol{\tau} \cdot \mathbf{X}^u(\mathbf{p}^{\boldsymbol{\tau}}, z)$, so period-1 disposable wealth is

$$y^1(z) \;=\; z - \boldsymbol{\tau} \cdot \mathbf{X}^u\big(\mathbf{p}^{\boldsymbol{\tau}}, z\big), \tag{5}$$

and the wealth carried into period 2 is

$$\omega(z) \;=\; z + rS^u(\mathbf{p}^{\boldsymbol{\tau}}, z). \tag{6}$$

In period 2 the agent consumes $\mathbf{X}^u(\mathbf{p}^{\boldsymbol{\tau}}, \omega(z))$ out of this wealth and pays $\boldsymbol{\tau} \cdot \mathbf{X}^u(\mathbf{p}^{\boldsymbol{\tau}}, \omega(z))$. Period-2 disposable wealth, the post-tax Lorenz object, is the wealth carried in net of the period-2 tax paid:

$$y^2(z) \;=\; \omega(z) - \boldsymbol{\tau} \cdot \mathbf{X}^u\big(\mathbf{p}^{\boldsymbol{\tau}}, \omega(z)\big) \;=\; z + rS^u(\mathbf{p}^{\boldsymbol{\tau}}, z) - \boldsymbol{\tau} \cdot \mathbf{X}^u\big(\mathbf{p}^{\boldsymbol{\tau}},\, z + rS^u(\mathbf{p}^{\boldsymbol{\tau}}, z)\big). \tag{7}$$

The period-1 tax enters y^2 through the taxed saving $S^u(\mathbf{p}^{\boldsymbol{\tau}}, z)$; the period-2 tax enters as the subtracted liability, evaluated at period-2 wealth $\omega(z)$ rather than at income z. Write $S_0(w) := S^u(\mathbf{p}, w)$ and $S_{\boldsymbol{\tau}}(w) := S^u(\mathbf{p}^{\boldsymbol{\tau}}, w)$ for saving at the two price vectors.

Proposition 1. *There exist a price vector $\mathbf{p} \in \mathbb{R}^k_{++}$, a utility $u \in \mathcal{U}$ satisfying Definition 2, a commodity tax $\boldsymbol{\tau} \in \mathbb{R}^k_+$ that is $(u, \mathbf{p})$-progressive with $\tau_i > 0$ on the luxury good, and a non-degenerate $\mathbf{z} \in \mathcal{Z}$ such that the tax increases second-period wealth inequality:*

$$\mathbf{y}^2(\mathbf{z}) \prec_L \mathbf{w}^2(\mathbf{z}).$$

A progressive commodity tax therefore need not achieve static inequality reduction in period 2 (Definition 5).

Proof. Steps 1–3 hold for any $\mathbf{p}$, u, $\boldsymbol{\tau}$ and $\bar{z}$ meeting the hypotheses; Step 4 exhibits a configuration that does.

Step 1: Period-2 wealth and the net response. The no-tax and post-tax period-2 wealth of an agent with income z are $w^2(z)$ and $y^2(z)$ of (4)–(7), and $g(z) = [y^2(z) - w^2(z)]/w^2(z)$ is the proportional change the tax induces. Both are strictly positive for every $z \in (0, \bar{z}\,]$: the solution to (1) is interior by Assumption 1, so $S^u > 0$ and hence $w^2, \omega > 0$, and by the period-2 budget identity $\mathbf{p}^{\boldsymbol{\tau}} \cdot \mathbf{X}^u + S^u = \omega$,

$$y^2 = \omega - \boldsymbol{\tau} \cdot \mathbf{X}^u = \mathbf{p} \cdot \mathbf{X}^u + S^u > 0.$$

Positivity is all the argument below requires of y^2.

Step 2: A differential net response reverses the ordering. Suppose $g(z_\ell) < g(z_h)$ for some $z_\ell < z_h$ in $(0, \bar{z}\,]$, and take the two-agent distribution $\mathbf{z} = (z_\ell, z_h) \in \mathscr{Z}$, which is non-degenerate. Writing $w^2_\ell = w^2(z_\ell)$ and so on, Lemma 1 gives $w^2_\ell < w^2_h$, and $y^2 = w^2(1 + g)$ gives

$$\frac{y^2_h}{w^2_h} = 1 + g(z_h) > 1 + g(z_\ell) = \frac{y^2_\ell}{w^2_\ell},$$

which cross-multiplied yields

$$y^2_h\, w^2_\ell - y^2_\ell\, w^2_h > 0. \tag{8}$$

Both distributions are then in ascending order: $w^2_\ell < w^2_h$ by Lemma 1, and $y^2_\ell < y^2_h$ because (8) and $w^2_\ell < w^2_h$ give $y^2_h\, w^2_\ell > y^2_\ell\, w^2_h > y^2_\ell\, w^2_\ell$.

Step 3: The Lorenz comparison. With two agents the only non-trivial cutoff is $m = 1$, at which the shares of the poorer agent are $y^2_\ell/(y^2_\ell + y^2_h)$ and $w^2_\ell/(w^2_\ell + w^2_h)$. All four levels are positive by Step 1, so

$$\frac{w^2_\ell}{w^2_\ell + w^2_h} - \frac{y^2_\ell}{y^2_\ell + y^2_h} = \frac{y^2_h\, w^2_\ell - y^2_\ell\, w^2_h}{\big(w^2_\ell + w^2_h\big)\big(y^2_\ell + y^2_h\big)} \;>\; 0$$

by (8). The shares coincide at $m = 2$, so $\mathbf{y}^2(\mathbf{z}) \prec_L \mathbf{w}^2(\mathbf{z})$, conditional on the supposition of Step 2. Note that no order-preservation condition on the period-2 liability is needed: only the positivity of Step 1 and the cross-product are used.

Step 4: The example utility. It remains to discharge the supposition of Step 2. Let u be the

additively separable power utility

$$u(x_1, \dots, x_k, s) = c_i\, x_i^{a} + c_s\, s^{a} + \sum_{j \neq i} c_j\, x_j^{\alpha_j}, \qquad 0 < \alpha_j < a < 1, \quad c_i, c_s, c_j > 0,$$

in which the luxury good x_i and saving share the exponent a and each remaining good carries a strictly smaller exponent. Being a sum of strictly increasing, strictly concave power functions, u is strictly increasing and strictly quasi-concave, so $u \in \mathcal{U}$. The shared exponent is what makes the mechanism operate. Dividing the first-order conditions for x_i and s,

$$\frac{s}{x_i} = \left(\frac{c_s\, p_i^{\tau}}{c_i}\right)^{\frac{1}{1-a}}, \qquad \text{so} \qquad \frac{\partial \ln(s/x_i)}{\partial \ln p_i^{\tau}} = \frac{1}{1-a} =: \epsilon > 1.$$

The taxed good and saving are substitutes, with an elasticity above one. Taxing x_i therefore drives the agent out of it and into saving. Example 1 exhibits a configuration of this family in which saving is a luxury, the tax is progressive, and $g(z_\ell) < g(z_h)$ at two endowments. Steps 2–3 then give $\mathbf{y}^2(\mathbf{z}) \prec_L \mathbf{w}^2(\mathbf{z})$ at the corresponding two-agent distribution, and a single such $\mathbf{z} \in \mathcal{Z}$ defeats the universal quantifier of Definition 5. ∎

Example 1 (A progressive commodity tax that reverses)**.** *Take the separable power utility*

$$u(x_i, x_j, s) = x_i^{3/4} + s^{3/4} + 4\, x_j^{1/2},$$

with one luxury good and one normal good, $a = \frac{3}{4}$ and $\alpha_j = \frac{1}{2}$, so the elasticity of substitution between the luxury good and saving is $\epsilon = 4$. Let $p_i = p_j = 1$ and impose a luxury tax alone, $\tau_i = 1$ and $\tau_j = 0$, so the taxed luxury price is $p_i^{\tau} = 2$, and take $r = 1$.

Writing μ for the reciprocal of the multiplier on the budget constraint, the first-order conditions give $s = (\frac{3}{4}\mu)^4$, $x_i = (\frac{3}{4}\mu/p_i^{\tau})^4$ and $x_j = (2\mu)^2$, so in the variable $v = \mu^2$ the budget is the quadratic

$$A\, v^2 + 4v = w, \qquad A = \tfrac{81}{256}\big(1 + (p_i^{\tau})^{-3}\big) = \begin{cases} \frac{81}{128} & \textit{untaxed}, \\ \frac{729}{2048} & \textit{under the tax}, \end{cases}$$

with saving $S = \frac{81}{256}v^2$. The saving rate $S(w)/w$ is strictly increasing at both price vectors, so saving is a luxury (Definition 2); the liability share $\tau_i X_i(\mathbf{p}^{\boldsymbol{\tau}}, w)/w$ is increasing, so $\boldsymbol{\tau}$ is $(u, \mathbf{p})$-progressive (Definition 8); and $S(w) < w$. Each holds at every wealth.

For two agents with endowments $\mathbf{z} = (5,\ 15)$*, writing* $S_0 = S^u(\mathbf{p}, z)$ *and* $S_\tau = S^u(\mathbf{p}^\tau, z)$*:*

	S_0	S_τ	*no-tax* w^2	*taxed* y^2	g
$z = 5$	0.361684	0.407819	5.361684	5.378405	+0.003119
$z = 15$	2.211952	2.784885	17.211952	17.554814	+0.019920

The tax raises saving at both endowments, and proportionally more at the higher one: by 12.8% *at* $z = 5$ *and by* 25.9% *at* $z = 15$*. Hence* $g(5) < g(15)$*, and Steps 2–3 of the proof apply: the bottom agent's share of second-period wealth falls from* 0.237520 *without the tax to* 0.234525 *with it,* $\mathbf{y}^2 \prec_L \mathbf{w}^2$*, though the tax is progressive throughout.*

The two periods need not give the same verdict on the same tax. In period 1, a progressive commodity tax compresses the distribution on which it is levied, whatever the preferences. By period 2, that distribution has been remade by the tax. The tax changes how much agents save, and the change can run the wrong way. Where the taxed good and saving are sufficiently close substitutes, the tax routes more of period-2 wealth through the channel that magnifies differences, and the compression it applies to the second period's purchases need not offset the dispersion it has induced in accumulation. Proposition 1 exhibits a preference and a tax for which it does not. Progressivity is not the failing. What fails is the assumption that a criterion verified at one date continues to hold at the next.

3.2 Income Tax

An income tax is levied on the endowment, before any consumption or saving decision is made, so it neither moves relative prices nor reaches accumulated wealth. Writing $\rho_{\mathsf{T}}(z) = z - \mathsf{T}(z)$ for the retention it leaves, the agent faces $\rho_{\mathsf{T}}(z)$ in place of z in each period, so period-1 post-tax wealth is $y^1(z) = \rho_{\mathsf{T}}(z)$ and period-2 post-tax wealth is

$$y^2(z) \;=\; \rho_{\mathsf{T}}(z) + r\, S^u\big(\mathbf{p}, \rho_{\mathsf{T}}(z)\big) \;=\; w^2\big(\rho_{\mathsf{T}}(z)\big), \tag{9}$$

the untaxed accumulation path of (4) evaluated lower down. The tax relocates the agent on that path rather than placing them on a new one, which is what distinguishes this instrument from a

commodity tax.

Definition 9 (Income Tax Schedule)**.** *An income tax schedule is a function* $\mathsf{T} : (0, \bar{z}\,] \to \mathbb{R}_+$ *with* $\mathsf{T}(z) \le z$*, continuous and continuously differentiable outside a finite set* K_{T} *of kink points, whose retention* $\rho_{\mathsf{T}}(z) = z - \mathsf{T}(z)$ *is strictly increasing. The schedule is* average-rate progressive *if the retention rate* $\rho_{\mathsf{T}}(z)/z$ *is non-increasing, equivalently if the average rate* $\mathsf{T}(z)/z$ *is non-decreasing.*

Where ρ_{T} is differentiable, write $\gamma_{\mathsf{T}}(z) = z\rho'_{\mathsf{T}}(z)/\rho_{\mathsf{T}}(z)$ for the *residual progression*, the elasticity of retention. Writing $m = \mathsf{T}'$ for the marginal rate and $t = \mathsf{T}/z$ for the average rate, $\rho'_{\mathsf{T}} = 1 - m$ and $\rho_{\mathsf{T}}/z = 1 - t$ give $\gamma_{\mathsf{T}} = (1 - m)/(1 - t)$, so $\gamma_{\mathsf{T}} \le 1$ if and only if $m \ge t$, which is average-rate progressivity in derivative form. The class contains the continuous piecewise-linear schedules that describe administered income tax systems: finitely many brackets, a constant marginal rate within each, no jump in liability at a threshold. Strictly increasing retention bounds the marginal rate below one in every bracket, so no bracket is steep enough to reverse two agents' order.

In period 1 the comparison is $\rho_{\mathsf{T}}(z)$ against z, exactly that of Jakobsson (1976) and Fellman (1976), and average-rate progressivity settles it. In period 2 it is between two points of the same accumulation path, and whether the ordering survives the composition is a further question. Both comparisons are instances of one criterion.

Lemma 2 (Ratio criterion)**.** *Let* $\phi, \psi : \mathcal{W} \to \mathbb{R}_{++}$ *be non-decreasing. Then*

$$\big(\phi(z_i)\big)_{i=1}^n \succeq_L \big(\psi(z_i)\big)_{i=1}^n \qquad \text{for every } \mathbf{z} \in \mathcal{Z}$$

if and only if $\phi(z)/\psi(z)$ *is non-increasing on* $(0, \bar{z}\,]$*, the range of endowments that* $\mathcal{Z}$ *visits. Dominance is strict at a given* $\mathbf{z}$ *if the ratio is strictly decreasing on its support. Sufficiency is due to Jakobsson (1976), Fellman (1976), and Kakwani (1977) for continuous distributions, with the discrete case in Le Breton et al. (1996); necessity follows from the two-level case.*[3]

[3]Fix $z_\ell < z_h$ in $(0, \bar{z}\,]$ and let $\mathbf{z} \in \mathcal{Z}$ place ξn agents at z_ℓ and $(1 - \xi)n$ at z_h, with ξn integral and $1 \le \xi n \le n - 1$. At the cutoff $m = \xi n$ the dominance inequality reads

$$\frac{\xi\,\phi(z_\ell)}{\xi\,\phi(z_\ell) + (1-\xi)\phi(z_h)} \;\ge\; \frac{\xi\,\psi(z_\ell)}{\xi\,\psi(z_\ell) + (1-\xi)\psi(z_h)}.$$

Proposition 2. *Let $u \in \mathcal{U}^\star$ and let T be average-rate progressive. Then T achieves static inequality reduction in period 2.*

Proof. Both y^2 and w^2 are continuous and strictly increasing on $(0, \bar{z}\,]$: S^u is strictly increasing by Lemma 1 and ρ_{T} by Definition 9. By Lemma 2 it therefore suffices that y^2/w^2 be non-increasing. The schedule is differentiable outside its finite set K_{T} of kink points, which partitions $(0, \bar{z}\,]$ into finitely many intervals; we compute on each and glue across the kinks at the end.

On such an interval, since $w^2(z) = z\big(1 + r\sigma(z)\big)$ and $z\sigma'(z) = \sigma(z)\big(\eta(z) - 1\big)$,

$$z\,\frac{d}{dz}\ln w^2(z) = \frac{1 + r\sigma(z)\eta(z)}{1 + r\sigma(z)} = 1 + \Phi(z),$$

and by (9) and the chain rule,

$$z\,\frac{d}{dz}\ln y^2(z) = \Big[1 + \Phi\big(\rho_{\mathsf{T}}(z)\big)\Big]\,\gamma_{\mathsf{T}}(z),$$

where $\gamma_{\mathsf{T}}(z) = z\rho'_{\mathsf{T}}(z)/\rho_{\mathsf{T}}(z)$. Subtracting,

$$z\,\frac{d}{dz}\ln\frac{y^2(z)}{w^2(z)} = \Big[1 + \Phi\big(\rho_{\mathsf{T}}(z)\big)\Big]\gamma_{\mathsf{T}}(z) \;-\; \big[1 + \Phi(z)\big]. \tag{10}$$

A non-negative tax gives $\rho_{\mathsf{T}}(z) \le z$, so admissibility (Definition 3) gives $\Phi\big(\rho_{\mathsf{T}}(z)\big) \le \Phi(z)$, and average-rate progressivity gives $\gamma_{\mathsf{T}}(z) \le 1$ against a bracket that exceeds one. Hence the right-hand side of (10) is non-positive on every interval of differentiability. The ratio y^2/w^2 is continuous on $(0, \bar{z}\,]$ and non-increasing on each of the finitely many intervals into which K_{T} divides it, hence non-increasing throughout. By Lemma 2, $\mathbf{y}^2(\mathbf{z}) \succeq_L \mathbf{w}^2(\mathbf{z})$ for every $\mathbf{z} \in \mathscr{Z}$. ∎

The two hypotheses divide the work, and neither is idle. Progressivity controls what the tax does to the endowment: it compresses the wealth the agent starts from, and $\gamma_{\mathsf{T}} \le 1$ is that statement in elasticity form. Admissibility controls where the tax leaves them: because it lowers wealth,

Both denominators are strictly positive, so cross-multiplying is order preserving; the common term $\xi^2\phi(z_\ell)\psi(z_\ell)$ cancels, leaving

$$\xi(1-\xi)\big[\phi(z_\ell)\,\psi(z_h) - \psi(z_\ell)\,\phi(z_h)\big] \;\ge\; 0.$$

Dividing by $\xi(1-\xi) > 0$ and by $\psi(z_\ell)\psi(z_h) > 0$ gives $\phi(z_\ell)/\psi(z_\ell) \ge \phi(z_h)/\psi(z_h)$. As $z_\ell < z_h$ were arbitrary in $(0, \bar{z}\,]$, the ratio is non-increasing there. Equivalently, in contrapositive form: if $\phi(z_\ell)/\psi(z_\ell) < \phi(z_h)/\psi(z_h)$ at some $z_\ell < z_h$, the same computation gives strict inequality of the opposite sign, so the two-level distribution at those points has $\big(\phi(z_i)\big)_i \prec_L \big(\psi(z_i)\big)_i$ at the cutoff $m = \xi n$.

the agent lands at a point of the accumulation path where the magnification is weaker, and (10) is non-positive on that account. A schedule can be progressive and still fail in period 2 if the preference is not admissible, since a magnification that is stronger lower down turns the tax's own relocation against it.

3.3 Wealth Tax

A wealth tax reaches what the other two instruments do not. A commodity tax works through relative prices and leaves the resource base untouched. An income tax lowers the base once, at the endowment, and what the agent then saves, together with the return on it, is never taxed: accumulated wealth bypasses the instrument. A wealth tax is levied on whatever an agent holds, so it applies twice over the two periods, to the endowment in period 1 and to endowment plus accumulated saving in period 2, and that accumulation is taxed along with the rest.

Writing $\rho_T(w) = w - T(w)$ for the retention the schedule leaves, period-1 post-tax wealth is $y^1(z) = \rho_T(z)$, the agent saves $S^u(\mathbf{p}, \rho_T(z))$ out of it, and the wealth they carry into period 2 is

$$w_T^2(z) \;=\; z + r\, S^u\big(\mathbf{p}, \rho_T(z)\big), \tag{11}$$

which the schedule then taxes again:

$$y^2(z) \;=\; \rho_T\big(w_T^2(z)\big). \tag{12}$$

The contrast with the income tax is exactly the outer application. There, post-tax wealth was $w^2(\rho_\mathrm{T}(z))$, the untaxed accumulation path evaluated at a lower endowment; here it is $\rho_T\big(w_T^2(z)\big)$, the same relocation followed by a second levy on what the relocation produced. Everything Proposition 2 established about the first effect carries over, and the second is what this subsection adds.

One consequence is worth recording before the definition. Because the schedule is evaluated at accumulated wealth as well as at the endowment, it must be defined on all of $\mathcal{W}$, not merely on the range $(0, \bar{z}\,]$ of endowments that an income tax ever sees. The second levy falls above $\bar{z}$

whenever accumulation carries an agent there, so progressivity is required on the whole operative range rather than on the endowment range alone.

Definition 10 (Wealth Tax Schedule)**.** *A wealth tax schedule is a function $T : \mathcal{W} \to \mathbb{R}_+$ with $T(w) \leq w$, continuous and continuously differentiable outside a finite set $K_T \subset \mathcal{W}$ of kink points, whose retention $\rho_T(w) = w - T(w)$ is strictly increasing. The schedule is* average-rate progressive *if the retention rate $\rho_T(w)/w$ is non-increasing on $\mathcal{W}$, equivalently if the average rate $T(w)/w$ is non-decreasing there.*

The domain matters. An income tax is levied once, on the endowment, and is never evaluated outside $(0, \bar{z}\,]$; a wealth tax is levied again on what accumulation produces, so both the schedule and the progressivity requirement extend to the whole operative range. Where ρ_T is differentiable, the residual progression is $\gamma_T(w) = w\rho_T'(w)/\rho_T(w)$, the elasticity of retention, and $\gamma_T \leq 1$ is average-rate progressivity in derivative form. The class contains the continuous piecewise-linear schedules of administered wealth taxes, though nothing below requires linearity within brackets: what the proofs use is that the kinks be finite in number, so that $\mathcal{W}$ splits into finitely many intervals on which the derivative exists.

In period 1 the two instruments coincide. No saving has yet occurred, so wealth is the endowment, the comparison is $\rho_T(z)$ against z, and a wealth tax levied on the endowment is an income tax levied on the same base. The characterization of Jakobsson (1976) and Fellman (1976) settles it: the tax reduces inequality in period 1 if and only if it is average-rate progressive, whatever the preferences. Saving does not enter, so nothing about u is used.

Period 2 is where the instruments part. The next proposition shows that on $\mathcal{U}^\star$ the wealth tax, like the income tax, reduces inequality in the second period.

Proposition 3. *Let $u \in \mathcal{U}^\star$ and let T be an average-rate progressive wealth tax (Definition 10). Then T achieves static inequality reduction in period 2.*

Proof. Write $w_0^2(z) = z + rS^u(\mathbf{p}, z)$ for the untaxed benchmark and w_T^2, y^2 as in (11)–(12). Both y^2 and w_0^2 are continuous and strictly increasing on $(0, \bar{z}\,]$: S^u is strictly increasing by Lemma 1 and

ρ_T by Definition 10. By Lemma 2 it therefore suffices that y^2/w_0^2 be non-increasing. The schedule is differentiable outside a finite set, which partitions $(0, \bar{z}\,]$ into finitely many intervals; we compute on each and glue across at the end.[4]

Step 1: saving rates per unit of endowment. Write $a(z) = \rho_T(z)/z$ for the retention rate, $\sigma(z) = S^u(\mathbf{p}, z)/z$ for the untaxed saving rate, and

$$q(z) \;=\; \frac{S^u\big(\mathbf{p}, \rho_T(z)\big)}{z}$$

for saving under the tax, again per unit of endowment. The two factors telescope,

$$a(z)\,\sigma\big(\rho_T(z)\big) = \frac{\rho_T(z)}{z} \cdot \frac{S^u(\mathbf{p}, \rho_T(z))}{\rho_T(z)} = \frac{S^u(\mathbf{p}, \rho_T(z))}{z} = q(z),$$

so $w_T^2(z) = z\big(1 + r\,q(z)\big)$ and $w_0^2(z) = z\big(1 + r\,\sigma(z)\big)$. The two economies differ in the rate at which the endowment is converted into accumulated wealth, and in nothing else at this stage. Write $\lambda_q = rq/(1+rq)$ and $\lambda_0 = r\sigma/(1+r\sigma)$ for the corresponding saving shares of period-2 wealth.

Step 2: the two accumulation elasticities. Differentiating $\ln q(z) = \ln S^u(\mathbf{p}, \rho_T(z)) - \ln z$ and using $\rho_T'(z) = \gamma_T(z)\rho_T(z)/z$,

$$z\,\frac{d}{dz}\ln q = \frac{\rho_T S^{u\prime}(\rho_T)}{S^u(\rho_T)} \cdot \frac{z\rho_T'}{\rho_T} - 1 = \varepsilon(z) - 1, \qquad \varepsilon(z) := \eta\big(\rho_T(z)\big)\,\gamma_T(z),$$

and likewise $z\,\frac{d}{dz}\ln\sigma = \eta(z) - 1$. Hence

$$z\,\frac{d}{dz}\ln w_T^2(z) = 1 + \lambda_q(z)\big(\varepsilon(z) - 1\big), \qquad z\,\frac{d}{dz}\ln w_0^2(z) = 1 + \Phi(z), \tag{13}$$

the second because $\lambda_0(\eta - 1)$ is the magnification of Definition 3.

Step 3: the second levy and the decomposition. Since $y^2(z) = \rho_T\big(w_T^2(z)\big) = a\big(w_T^2(z)\big)\,w_T^2(z)$ and $a'(w)/a(w) = \big(\gamma_T(w) - 1\big)/w$, the chain rule gives

$$z\,\frac{d}{dz}\ln y^2 = \Big[\gamma_T\big(w_T^2(z)\big) - 1\Big]\,z\,\frac{d}{dz}\ln w_T^2 \;+\; z\,\frac{d}{dz}\ln w_T^2 = \gamma_T\big(w_T^2(z)\big)\cdot z\,\frac{d}{dz}\ln w_T^2.$$

[4]The exceptional set is larger than the schedule's own kinks. The schedule is applied twice, at z and at $w_T^2(z)$, so γ_T enters the derivative evaluated at both points and the composite fails to be differentiable at any z with $w_T^2(z) \in K_T$. The set of such endowments is $(w_T^2)^{-1}(K_T)$, finite because w_T^2 is strictly increasing, and these are ordinary points of the tax code: an agent whose endowment lies well inside a bracket may accumulate to a threshold. The relevant set is therefore $E = K_T \cup (w_T^2)^{-1}(K_T)$. The income tax has no counterpart, being levied once, at z.

Subtracting the second identity of (13) and adding and subtracting $1 + \lambda_q(\varepsilon - 1)$,

$$z\,\frac{d}{dz}\ln\frac{y^2(z)}{w_0^2(z)} = \underbrace{\Big[\gamma_T\big(w_T^2(z)\big) - 1\Big]\Big[1 + \lambda_q(\varepsilon - 1)\Big]}_{\text{(I): the second levy}} + \underbrace{\lambda_q(z)\big(\varepsilon(z) - 1\big) - \Phi(z)}_{\text{(II): the accumulation channel}}{}^{5}. \tag{14}$$

Step 4: both terms are non-positive. For (I): average-rate progressivity gives $\gamma_T \le 1$ at every wealth in $\mathcal{W}$, in particular at $w_T^2(z)$, which lies in $\mathcal{W}$ by Definition 1. The second bracket is the elasticity $z\,\frac{d}{dz}\ln w_T^2$ of a strictly increasing function, hence positive. So (I) ≤ 0.

For (II): if $\varepsilon(z) \le 1$ the first term is non-positive while $\Phi(z) > 0$ by luxury saving, so (II) < 0. If $\varepsilon(z) > 1$, two bounds apply. A non-negative tax gives $\rho_T(z) \le z$, so $q(z) = S^u(\mathbf{p}, \rho_T(z))/z \le S^u(\mathbf{p}, \rho_T(z))/\rho_T(z) = \sigma\big(\rho_T(z)\big)$, and since $x \mapsto rx/(1 + rx)$ is increasing, $\lambda_q(z) \le \lambda\big(\rho_T(z)\big)$; and $\gamma_T(z) \le 1$ gives $\varepsilon(z) - 1 \le \eta\big(\rho_T(z)\big) - 1$. Multiplying the two,

$$\lambda_q(z)\big(\varepsilon(z) - 1\big) \;\le\; \lambda\big(\rho_T(z)\big)\Big[\eta\big(\rho_T(z)\big) - 1\Big] \;=\; \Phi\big(\rho_T(z)\big) \;\le\; \Phi(z),$$

the last step by admissibility, since $\rho_T(z) \le z$. So (II) ≤ 0.

The right-hand side of (14) is therefore non-positive on every interval of differentiability. The ratio y^2/w_0^2 is continuous on $(0, \bar{z}\,]$ and non-increasing on each of the finitely many intervals into which E divides it, hence non-increasing throughout. By Lemma 2, $\mathbf{y}^2(\mathbf{z}) \succeq_L \mathbf{w}_0^2(\mathbf{z})$ for every $\mathbf{z} \in \mathcal{Z}$. ∎

Here too the hypotheses divide the work, but progressivity now presses twice. Admissibility governs where the tax leaves the agent: it lowers their wealth, so they save at a point on the accumulation path where the magnification is weaker, which signs term (II). Progressivity governs how hard the schedule presses. At the endowment it compresses what the agent saves out of. At accumulated wealth it compresses again, and that second pressing is term (I).

The second pressing explains why Definition 10 asks for progressivity over the entire operative

[5]The right-hand side of (14) is an exact derivative, so its sign characterizes period-2 reduction completely: the tax is period-2 inequality reducing if and only if it is non-positive throughout $(0, \bar{z}\,]$. The hypotheses of the proposition are the sufficient core obtained by signing the two terms separately, and they are stronger than necessary. The slack is term (I), strictly negative wherever the schedule is strictly progressive at period-2 wealth, which leaves room for term (II) to be positive without overturning the conclusion. The slack vanishes as the schedule becomes asymptotically proportional at the top, where $\gamma_T \to 1$.

range, since second-period wealth can exceed $\bar{z}$. It is also what separates the two instruments. An income tax acts once, and the criterion is (10); the wealth tax acts again on what the first action produced, and (14) adds term (I) to record it. Both terms point the same way on $\mathcal{U}^\star$, so the wider reach never works against the instrument. The next proposition shows that it works for it.

Definition 11 (Marginal-Rate Progressive Schedule)**.** *A schedule f is* marginal-rate progressive *if its marginal rate f' is non-decreasing, equivalently if f is convex.*

Proposition 4. *Let $u \in \mathcal{U}$ satisfy Definition 2 and let $f : \mathcal{W} \to \mathbb{R}_+$ be a marginal-rate progressive schedule, levied in one economy on income and in another on wealth. Then post-tax wealth under the wealth tax Lorenz dominates post-tax wealth under the income tax in both periods: the two distributions coincide in period 1, and in period 2*

$$\mathbf{y}^2_{wealth}(\mathbf{z}) \succeq_L \mathbf{y}^2_{income}(\mathbf{z}) \qquad \forall\, \mathbf{z} \in \mathcal{Z}.$$

Proof. In period 1 no saving has yet occurred, so wealth is the endowment and both instruments levy f on z. Post-tax wealth is $\rho_f(z)$ under each, and the two distributions coincide.

Period 2 is where the instruments part. Each agent retained $\rho_f(z)$ in period 1 under either instrument, so they saved the same amount and carry the same gross wealth into period 2 in the two economies: $w(z) = z + r\, S^u\big(\mathbf{p}, \rho_f(z)\big)$. What differs is the base. The income tax only reaches the endowment, leaving $A(z) = w(z) - f(z)$. The wealth tax reaches the whole stock, leaving $B(z) = w(z) - f\big(w(z)\big)$. Both are continuous and strictly increasing on $(0, \bar{z}\,]$: ρ_f is strictly increasing by hypothesis, S^u by Lemma 1, and w as a composition of the two. By Lemma 2 it therefore suffices that B/A be non-increasing.

The gap between them is the extra liability the wider base creates, $D(z) = f\big(w(z)\big) - f(z)$, so that $B/A = 1 - D/A$ and the question is whether D/A rises. It does, and the reason separates into two factors. Write

$$\bar{m}(z) = \frac{f\big(w(z)\big) - f(z)}{w(z) - z}$$

for the average marginal rate on the wealth the agent accumulates. Since that wealth is exactly what

saving carries forward, $w(z) - z = rS^u(\mathbf{p}, \rho_f(z))$, and since $A(z) = \rho_f(z) + rS^u(\mathbf{p}, \rho_f(z))$,

$$\frac{D(z)}{A(z)} = \bar{m}(z)\,\lambda\big(\rho_f(z)\big), \qquad \lambda(x) = \frac{r\,\sigma(x)}{1 + r\,\sigma(x)}. \tag{15}$$

The two factors have different sources. The first, $\bar{m}$, belongs to the schedule: it is the rate the tax code charges on what the agent accumulates, and it moves only if the rate structure does. The second, $\lambda(\rho_f(z))$, belongs to preferences: it is the share of period-2 wealth that comes from saving, and luxury saving makes it rise with wealth whatever the tax.

Marginal-rate progressivity makes f convex, and for a convex function the chord slope $Q(a, b) = \big(f(b) - f(a)\big)/(b - a)$ rises when either a or b increases. Now $\bar{m}(z) = Q\big(z, w(z)\big)$, and both endpoints increase with z, the second by Lemma 1. Raising first one and then the other gives $\bar{m}(z_1) \le \bar{m}(z_2)$ whenever $z_1 < z_2$. [6] Both factors in (15) are positive and non-decreasing, and one is strictly increasing, so D/A rises and B/A falls. Lemma 2 then gives $\mathbf{y}^2_{\text{wealth}} \succeq_L \mathbf{y}^2_{\text{income}}$, strictly at any non-degenerate $\mathbf{z}$. ■

Given the same convex schedule, then, the wealth tax leaves a more equal distribution than the income tax in every period. The advantage is exactly the wider base: the wealth tax reaches what the agent has accumulated, and luxury saving makes accumulation a rising share of what agents hold.

4 Dynamic Inequality

The previous section compared each period's distribution with what it would have been under no tax. That comparison is static: it asks whether the tax compresses the distribution at a date, and it asks the question afresh at each date. A tax can pass that test twice and still leave a distribution

[6] Marginal-rate progressivity cannot be weakened to average-rate progressivity. Both instruments leave the agent the same gross wealth $w(z)$, so by (15) the wealth tax's advantage over the income tax turns entirely on $\bar{m}$, the rate the code charges on what the agent accumulates. Average-rate progressivity permits marginal rates that fall, so that rate can be lower for a richer agent: if accumulation carries them across a threshold into a cheaper bracket, the wider base costs them proportionally less than it costs a poorer agent. The wealth tax then loses ground where it should gain it, B/A rises rather than falls, and the ranking of the two instruments reverses. Convexity rules this out by keeping $\bar{m}$ non-decreasing.

dispersed over time, as long as the second-period distribution is less compressed than the first. Inequality reduction at a given date and inequality reduction over time are distinct properties, and the former does not imply the latter.

This section asks the second question. The object is no longer a distribution against its untaxed counterpart but the sequence itself: does each period's post-tax distribution Lorenz dominate the one before it? In the untaxed economy the answer follows from luxury saving alone. The rich save a larger share of what they hold, so they carry proportionally more into the next period, and the distribution pulls apart whatever the prices and whatever the return (Proposition 5).

What a tax can do about that depends on what it reaches. An income tax is levied on the endowment, so it rescales what the agent starts with and leaves the subsequent accumulation untouched; the sequence diverges under every income tax (Remark 3). A wealth tax reaches accumulated wealth itself, and is the only instrument here that can compress the sequence. But a wealth tax schedule can be average-rate progressive throughout and the sequence still diverge (Section 4.2). What static redistribution secures at each date, it does not thereby secure across dates.

Definition 12 (Dynamic Inequality Reduction)**.** *A sequence of post-tax wealth distributions* $\{\mathbf{y}^t(\mathbf{z})\}_{t\geq 1}$ *exhibits* dynamic inequality reduction *if each period's distribution Lorenz dominates the one before it:*

$$\mathbf{y}^{t+1}(\mathbf{z}) \succeq_L \mathbf{y}^t(\mathbf{z}) \qquad \forall\, t \geq 1,\ \forall\, \mathbf{z} \in \mathscr{Z}.$$

The comparison is internal to the taxed economy: period $t+1$ against period t, with no untaxed benchmark. Each comparison is a Lorenz comparison, so by Atkinson (1970) it commands the assent of every inequality-averse planner, whatever the form of that aversion, and the criterion inherits its standing from the one the classical literature adopts. This is what separates it from Definition 5, which compares the taxed economy with the untaxed one at a single date. A tax may satisfy the one and fail the other, and the results below turn on exactly that gap.

We state Definition 12 for a general horizon, since it formalizes a notion of inequality dynamics that does not depend on the number of periods analyzed. The results concern the two-period case,

$t = 1$. Nothing in the mechanism is special to it: each period after the first has the same structure, with income received afresh, saved out of post-tax wealth, and carried forward, so the saving rate governs the step from w^t to w^{t+1} exactly as it governs the step from w^1 to w^2. We do not pursue the general-horizon statements here.

Proposition 5. *In the absence of taxation, wealth inequality strictly increases: for every $u \in \mathcal{U}$ satisfying Definition 2 and every non-degenerate $\mathbf{z} \in \mathcal{Z}$,*

$$\mathbf{w}^2(\mathbf{z}) \prec_L \mathbf{w}^1(\mathbf{z}).$$

Proof. Period-1 wealth is the endowment, $w^1(z) = z$, and period-2 wealth is $w^2(z) = z + rS^u(\mathbf{p}, z) = z\big(1 + r\sigma(z)\big)$. Both are continuous and strictly increasing, by Lemma 1. Their ratio is

$$\frac{w^1(z)}{w^2(z)} = \frac{1}{1 + r\,\sigma(z)},$$

strictly decreasing in z, because luxury saving (Definition 2) makes the saving rate σ strictly increasing. Applying Lemma 2 with $\phi = w^1$ and $\psi = w^2$ gives $\mathbf{w}^1 \succ_L \mathbf{w}^2$, strictly since $\mathbf{z}$ is non-degenerate and the ratio is strictly decreasing across its support. ∎

The saving rate is the sole factor behind the divergence. Period-2 wealth is $z(1 + r\sigma(z))$, so a rising saving rate makes it grow faster than proportionally. Neither the price vector nor the size of the return affects this. The divergence is driven solely by luxury saving, which is why no choice of prices, returns, or starting distributions can reverse its direction. Only a departure from luxury saving could.

4.1 Income Tax

An income tax is levied on the endowment, so it changes what the agent starts with and nothing about what happens next. Post-tax wealth in period 1 is $y^1(z) = \rho_{\mathsf{T}}(z)$, the agent saves out of it, and accumulation is untaxed, so

$$y^2(z) = \rho_{\mathsf{T}}(z) + r\,S^u\big(\mathbf{p}, \rho_{\mathsf{T}}(z)\big) = \rho_{\mathsf{T}}(z)\Big(1 + r\,\sigma\big(\rho_{\mathsf{T}}(z)\big)\Big).$$

Remark 3. *No income tax schedule achieves dynamic inequality reduction: for every $u \in \mathcal{U}$ satisfying Definition 2, every T as in Definition 9, and every non-degenerate $\mathbf{z} \in \mathcal{Z}$,*

$$\mathbf{y}^2(\mathbf{z}) \;\prec_L\; \mathbf{y}^1(\mathbf{z}).$$

Proof. Both y^1 and y^2 are continuous and strictly increasing, by Definition 9 and Lemma 1. Their ratio is

$$\frac{y^1(z)}{y^2(z)} = \frac{1}{1 + r\,\sigma\big(\rho_{\mathsf{T}}(z)\big)},$$

strictly decreasing in z, since σ is strictly increasing by luxury saving and ρ_{T} is strictly increasing. Applying Lemma 2 with $\phi = y^1$ and $\psi = y^2$ gives $\mathbf{y}^1 \succ_L \mathbf{y}^2$. ∎

The result holds for every schedule, regardless of its rate structure, and uses only luxury saving. An income tax enters only through the argument of σ, so it can move the agent down the saving-rate curve but cannot alter the curve, and a saving rate that rises in wealth rises in post-tax wealth too. A progressive income tax passes the static test in period 1, passes it again in period 2, and still lets wealth grow more unequal across them. Static inequality reduction at every date is not dynamic inequality reduction, and the income tax shows it most cleanly, being the instrument whose static credentials are least in doubt.

4.2 Wealth Tax

The income tax failed for a structural reason: entering only through the argument of the saving rate, it could move the agent down the accumulation path but could not alter the path. A wealth tax is not so confined. It is levied again on what accumulation produced, so the second application acts directly on the dispersion the first one could not touch, and the ratio that governs the sequence is no longer of the form $1 + r \times$ (a saving rate). Writing $y^1(z) = \rho_T(z)$ and $y^2(z) = \rho_T\big(w_T^2(z)\big)$ with w_T^2 as in (11), dynamic reduction asks that

$$\frac{y^2(z)}{y^1(z)} = \frac{\rho_T\big(w_T^2(z)\big)}{\rho_T(z)}$$

be non-increasing, and the numerator now carries a levy of its own. The wealth tax is thus the only instrument considered here with the reach to compress the sequence rather than merely each term of it.

Whether it does is a further question, and the tax code alone does not settle it. Two forces meet. Accumulation disperses, and the second levy compresses what accumulation dispersed. Which prevails depends on how agents save, not only on the schedule. The example below shows that the compression can lose, and it is not a pathological case. The preference is admissible, the schedule is average-rate progressive across the whole operative range, and the tax reduces inequality in each period taken separately. Every hypothesis the static results use is satisfied. What fails is the passage from one date to the next.

Example 2 (A progressive wealth tax that fails dynamically)**.** *Let saving have constant elasticity,* $S(w) = c\,w^{\beta}$ *with* $c = \frac{1}{10}$ *and* $\beta = \frac{3}{2}$*, so the saving rate* $\sigma(w) = \frac{1}{10}\sqrt{w}$ *is strictly increasing and saving is a luxury. The expansion path* $w \mapsto (w - S(w), S(w))$ *is strictly increasing in both coordinates for* $w < (c\beta)^{-1/(\beta-1)} = 44.44$*, so on that range it is rationalized by some* $u \in \mathcal{U}$*. Let* T *be the wealth tax with an exemption at* 1 *and a marginal rate of* 20%*, so* $\rho_T(w) = \frac{4}{5}w + \frac{1}{5}$ *above the exemption, and take* $r = 1$*. The retention rate* $\rho_T(w)/w$ *is non-increasing, so* T *is average-rate progressive throughout.*

For two agents with endowments $\mathbf{z} = (2, 4)$*, both in the taxed bracket:*

	$\rho_T(z)$	$w_T^2(z)$	$y^2(z)$	$\gamma_T(z)$ / $\gamma_T(w_T^2)$
$z = 2$	1.800000	2.241495	1.993196	0.888889 / 0.899659
$z = 4$	3.400000	4.626929	3.901543	0.941176 / 0.948738

The tax compresses at every point, $\gamma_T < 1$ *throughout, and it reduces inequality in each period separately: the bottom agent's share rises from* 0.333333 *to* 0.346154 *in period 1 and from* 0.322306 *to* 0.338131 *in period 2, so static inequality reduction holds at both dates. Yet across the two, the bottom share falls from* 0.346154 *to* 0.338131*:* $\mathbf{y}^2 \prec_L \mathbf{y}^1$*, and dynamic inequality reduction fails.*

The mechanism is visible in the criterion. Dynamic reduction asks that y^2/y^1 be non-increasing,

and with $y^1 = \rho_T(z)$ and $y^2 = \rho_T\big(w_T^2(z)\big)$,

$$z\,\frac{d}{dz}\ln\frac{y^2(z)}{y^1(z)} = \gamma_T\big(w_T^2(z)\big)\Big[1 + \lambda_q(z)\big(\varepsilon(z) - 1\big)\Big] - \gamma_T(z),$$

in the notation of the proof of Proposition 3. Two forces compound on the right. The second levy transmits at rate $\gamma_T(w_T^2)$, compressing; accumulation transmits at $1 + \lambda_q(\varepsilon - 1)$, magnifying, since saving is a luxury. At $z = 4$ the accumulation factor is 1.0558 and the levy factor 0.9487, so their product is 1.0017 against $\gamma_T(z) = 0.9412$: the compression applied to accumulated wealth does not undo the dispersion accumulation produced. Because γ_T rises within a bracket, the compression is weaker at the higher wealth than at the lower, which works in the same direction.

The configuration is not a knife-edge. A legible sufficient condition is $\beta > 1/\gamma_T(z)$, which makes the accumulation factor exceed one on its own; here $\beta = 1.5$ against thresholds of 1.125 at $z = 2$ and 1.063 at $z = 4$. The condition is far from necessary, since $\gamma_T\big(w_T^2\big) > \gamma_T(z)$ supplies the remainder, so divergence survives well below it.

Remark 4. *Static inequality reduction at every date does not imply dynamic inequality reduction, and the gap is not confined to unusual preferences or unusual schedules. In Example 2 saving is a luxury, the schedule is average-rate progressive throughout, and the tax reduces inequality in each period taken separately. The sequence diverges nonetheless.*

This identifies a criterion, not an impossibility. Average-rate progressivity was derived for the static problem and settles it completely: it is exactly what compresses a distribution at the date it is levied. The dynamic problem is a different comparison, between consecutive post-tax distributions rather than between a taxed and an untaxed one at a single date. There is no reason a criterion built for the static comparison should settle the dynamic one, and the example confirms that it does not. What governs the dynamic comparison is the second levy's compression against the dispersion accumulation produces, which the criterion above states exactly. A schedule designed against it would be progressive in a dynamic sense, a notion the static theory neither supplies nor rules out.

5 Conclusion

This paper introduces dynamic inequality reduction, the requirement that each period's post-tax distribution be at least as equal as the one before, and asks whether progressive taxation can achieve it when saving is a luxury good. Without taxation it cannot hold: because the rich save a larger share of what they have, the distribution diverges (Proposition 5). No income tax stops the divergence, whatever its rates, since it moves agents down the saving-rate curve without changing the curve (Remark 3). A wealth tax is the only instrument considered here that can, because it is levied again on what accumulation has produced, but it need not: Example 2 gives a progressive wealth tax that reduces inequality in each period while the sequence still diverges. Reducing inequality at every date and reducing it over time are different properties (Remark 4), and this is not a failure of the classical theory, which answers a different comparison. Date by date, its characterizations extend to income and wealth taxes but not to commodity taxes (Propositions 2, 3, and 1), and a wealth tax is more equalizing than an income tax levied through the same marginal-rate progressive schedule (Proposition 4).

Three simplifications deserve comment. The return is common across agents, so divergence runs through the saving-rate channel alone; returns also rise with wealth (Fagereng et al., 2020), a force that compounds the first rather than offsetting it. The return is also exogenous, and a wealth tax in general equilibrium would alter it. The divergence and income-tax results hold for any fixed return; how an endogenous return would interact with the wealth tax is left open. Finally, the mechanism is exhibited over two periods, though nothing in it is particular to two: income arrives afresh, is taxed, and is saved out of what remains, in every period alike.

A fourth assumption is of a different character, and it is worth being explicit about what it buys. The agents here are myopic, saving out of current wealth and treating the future levy as given. That is what keeps each period's budget set linear, and it is what makes the average-rate criterion the relevant one. Forward-looking agents face a different problem. They know that saving pushes them into a higher bracket, so the return they earn is net of the marginal rate there. The intertemporal budget set is then kinked, and agents bunch at the thresholds. What governs behavior

is the marginal rate, not the average. Work in progress takes up that problem.

A tax designed for the dynamic problem would have to take back, through its second levy, enough of what accumulation has spread apart. Such a schedule would be progressive in a sense the static theory neither supplies nor rules out, and finding the rule that identifies these schedules is the natural next question.

Wealth concentration has risen across the developed world for four decades, through repeated rounds of progressive taxation. The model does not claim to explain that record, and many forces bear on it. But it identifies one mechanism the record is consistent with: a tax can reduce inequality at every date while inequality still grows. Bending the trajectory is a different problem.

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